Reliability Analysis of Timber Elements Under Different Load Types and Identification of Critical Scenarios for the Evaluation of Existing Structures

Maria Loebjinski, Wolfgang Rug, and Hartmut Pasternak

Abstract This contribution presents reliability analyses of structural members made from timber under different load combinations relevant for common structures. The study embraces members under strain induced by permanent action and live load, as well as permanent action and snow/wind load. Snow and wind load are superimposed applying the Ferry Borges and Castanheta load combination rule. Results are analysed to identify critical design situation from a statistical point of view assuming a one-hundred percent workload of the semi-probabilistic design situation considering partial safety factors from current design codes. Studies are performed for a reference period of 50 years. The studies show that small rooms under high load fluctuation (live load) are critical in terms of calculated reliability. What is more, a high load share of snow load is also particular critical. Thus, flat roofs are to be investigated with certain care. Results are used to identify and classify design situation for modification of partial safety factors on the material side. In this respect, different design situations can be treated more optimal and thus load-bearing capacities in existing timber structure may be activated and considered if available.

Keywords Reliability analysis · Timber structures existing structures · Code calibration

1 Introduction

The investigation and evaluation of existing structures are important and challenging tasks for practicing engineers. However, in times of sustainable economy providing solutions with less consumption of energy and resources moving more and more

M. Loebjinski (✉)
Technische Universität Braunschweig, Braunschweig, Germany
e-mail: Maria.Loebjinski@b-tu.de

W. Rug
BaSys GmbH, Lenzen, Germany

H. Pasternak
Brandenburg University of Technology, Cottbus, Germany

© The Author(s), under exclusive license to Springer Nature Switzerland AG 2021

J. C. Matos et al. (eds.), *18th International Probabilistic Workshop*, Lecture Notes in Civil Engineering 153, https://doi.org/10.1007/978-3-030-73616-3_49

into the focus of common attention, building with existing structures experiences a significant increase in prosperity. The identification of critical members and their qualified investigation are central parts of the evaluation of the load-bearing capacity and serviceability of a structure and thus their substance-careful maintenance and, if necessary, rehabilitation.

This contribution presents reliability analyses of timber members under typical loadings. These are uniaxial strain from permanent action and one or two time-variant loads, two-axial strain from bending and compression and two-axial bending. The influence of slenderness has not been considered.

Reliability analyses have been performed applying First Order Reliability Method (FORM) in MATLAB. The reliability level that is reached applying the semi-probabilistic safety format of EN 1990:2010-12 [1] and EN 1995-1-1:2010-12 [2] is analysed for a variation of input parameters, i.e. load ratios and coefficient of variation (cov) of the material strength. Sensitivity analyses have been performed for a better understanding of the impact of basic variables on the probability of failure and safety index respectively. Based on an improved knowledge of sensitivities, strengthening measures can be carried out for critical elements from a statistical point of view to realize a substance-careful redevelopment of a structure in service.

What is more, based on these calculation an estimate of the implicit safety level of current design rules in timber engineering can be made. This can serve as a basis for an adjustment of partial safety factors for existing structures to be used for an evaluation applying the semi-probabilistic safety concept of current codes.

2 Load Scenarios, Model Assumptions and Limit State Functions

2.1 Load Scenarios

The following five scenarios have been analysed:

1. Uniaxial stress from permanent action and live load
2. Uniaxial stress from permanent action, snow load and wind load
3. Two-axial bending from permanent action, snow load and wind load
4. Compression from permanent action and snow load, bending from wind load (combination of stresses without considering the influence of slenderness)
5. Compression from permanent action and live load, bending from wind load (combination of stresses without considering the influence of slenderness)

2.2 Model Assumptions and Basic Variables

Table 1 illustrates probabilistic parameters as applied. In all limit state functions

Table 1 Probabilistic parameters

	Variable	Sym	Distr.	μ_X	V_X	Notes
Resistance	*Timber*					
	Bending	R_m	LN	1.0	0.25	Based on [4]
	Compression parallel to grain	R_c	LN	1.0	0.20	
	Tension parallel to grain	R_t	LN	1.0	0.30	
Loads	*Permanent action*	G	N	1.0	0.10	Based on [5]
	Live load					
	Distribution of maxima in reference period					
	Small room (≤20 m^2)	N	GUM	1.0	0.40	Based on studies applying [5–7]
	Large room (>20m^2)	N		1.0	0.25	
	Presence period in reference period (days)	n_p	det.	50 · 365	–	Based on [3]
	Load changes in presence period	n_r	det.	5 · 365	–	
	Snow load					
	Distribution of maxima in reference period	S	GUM	1,0	0.25	Based on [8]
	Presence period in reference period (days)	n_p	det.	50 · 60	–	Based on [3] (adjusted for GER)
	Load changes in presence period	n_r	det.	10	–	
	Wind load					
	Distribution of maxima in reference period	W	GUM	1.0	0.16	Based on [7, 8]
	Instantaneous value of wind load	W_{mom}	GUM	0.16	1.00	Based on [9]
	Presence period in reference period (days)	n_p	det.	50 · 365	–	Based on [3]
	Load changes in presence period	n_r	det.	50 · 365	–	
Model	*Resistance side*					

(continued)

Table 1 (continued)

	Variable	Sym	Distr.	μ_x	V_x	Notes
	Qualified survey in situ required	θ_R	N	1.0	0.07	Suggestion
	Load side					
	Permanent actions, qualified survey required	θ_G	N	1.0	0.05	Suggestion
	Live load	θ_N	N	1.0	0.10	Based on [10–12]
	Snow load	θ_S	N	1.0	0.10	
	Wind load	θ_W	N	1.0	0.10	

(LSF) a design parameter z_d has been introduced ensuring a one-hundred percent utilization of the semi-probabilistic design equation, see e.g. [3]. According to [4] R_m is correlated with R_c and R_t respectively by $\rho = 0.8$. Snow and wind load have been assumed uncorrelated, which is a simplification. For a more detailed analysis, modeling applying stochastic processes would be required. All calculations have been performed considering current values of partial safety factors (PSF) ($\gamma_G = 1.35, \gamma_Q = 1.5, \gamma_M = 1.3$).

2.3 *Limit State Functions*

2.3.1 Limit State Function and Design Parameter for Uniaxial Stress from Permanent Action and One Time-Variant Load

The LSF applied for load scenario (1) is

$$g = z_d \cdot k_{\text{mod}} \cdot R_i \cdot \theta_R - LV_G \cdot S_G \cdot \theta_{S,G} + (1 - LV_G) \cdot S_{Q1} \cdot \theta_{S,Q1} \tag{1}$$

with

$$z_d = \frac{\left(LV_G \cdot \gamma_G \cdot g_k + (1 - LV_G) \cdot \gamma_Q \cdot q_{k,N}\right) \cdot \gamma_M}{k_{\text{mod}} \cdot f_{k,m}} \tag{2}$$

Denotations of variables are defined in Table 1. Additionally, LV_G is the load ratio of the permanent actions of the total load, γ_G, γ_Q and γ_M are PSF for permanent action, variable action and material resistance respectively, g_k is the characteristic value (expected value) of the permanent action, $q_{k,N}$ is the characteristic value of the live load (model value as T_{ref}= *50a*), $f_{k,m}$ is the characteristic value of the material resistance (5%-quantile), and k_{mod} is the modification factor considering load duration and moisture content.

2.3.2 Limit State Function and Design Parameter for Uniaxial Stress from Permanent Action and Two Time-Variant Loads

Applying the load combination rule of Ferry Borges and Castanheta [13] the LSF (scenario 2) is

$$g = z_d \cdot k_{\text{mod}} \cdot R \cdot \theta_R - \big(LV_G \cdot S_G \cdot \theta_{S,G} + (1 - LV_G) \\ \cdot \big(LV_{Q1} \cdot S_{Q,1}^{n_1} \cdot \theta_{S,1} + \big(1 - LV_{Q2}\big) \cdot S_{Q,2}^{n_2} \cdot \theta_{S,2}\big)\big) \quad (3)$$

with

$$n_1 = \frac{\min(n_{p1}, n_{p2})}{n_{p1}} \quad (4)$$

$$n_2 = \frac{\max(d_1, d_2)}{n_{p2}} \quad (5)$$

$$d_1 = \frac{n_{p1}}{n_{r1}} \quad (6)$$

$$d_2 = \frac{n_{p2}}{n_{r2}} \quad (7)$$

Variables are explained in Table 1. Additionally, LV_{Q1} is the load ratio of the first variable load referring to the whole amount variable loads. The design parameter is

$$z_d = \frac{\gamma_M}{k_{\text{mod}} \cdot f_{k,m}} \\ \cdot \big(LV_G \cdot \gamma_G \cdot g_k + LV_Q \cdot \big(LV_{Q1} \cdot \gamma_Q \cdot q_{k,1} + LV_{Q2} \cdot \gamma_Q \cdot \psi_{Q2} \cdot q_{k,2}\big)\big) \quad (8)$$

where ψ_{Q2} is the combination factor for the accompanying load. Here, k_{mod}= 1.0. Applying Turkstras [14] load combination the LSF is

$$g = z_d \cdot k_{\text{mod}} \cdot S \cdot R \cdot \theta_R - LV_G \cdot S_G \cdot \theta_{S,G} \\ + \big((1 - LV_G) \cdot LV_{Q,1} \cdot S_{Q,1} \cdot \theta_{S,1} + \big(1 - LV_{Q,1}\big) \cdot S_{Q,2,mom} \cdot \theta_{S,2}\big) \quad (9)$$

with

$$z_d = \frac{\gamma_M}{k_{\text{mod}} \cdot f_k} \cdot (LV_G \cdot \gamma_G \cdot g_k + (1 - LV_G) \\ \cdot \big(LV_{Q1} \cdot \gamma_Q \cdot q_{k,1} + \big(1 - LV_{Q1}\big) \cdot \gamma_Q \cdot \psi_{Q2} \cdot q_{k,2}\big)\big) \quad (10)$$

where additionally to the variables explained above, LV_{Q1} is he load ratio of the first variable load related to the total variable load.

2.3.3 Limit State Function and Design Parameter for Two-Axial Bending

The LSF for load scenario (6) is

$$g = 1 - \left(\frac{1}{z_{d,z}} \cdot \frac{LV_G \cdot S_{G,z} \cdot \theta_{S,G} + (1 - LV_G) \cdot LV_{Q,1} \cdot S_{Q,1,z}^{n_1} \cdot \theta_{S,1}}{R_m} + \frac{1}{z_{d,y}} \cdot \frac{(1 - LV_G) \cdot \left(1 - LV_{Q,1}\right) \cdot S_{Q,2,z}^{n_2} \cdot \theta_{S,2}}{R_m} \right) \cdot \frac{1}{\theta_{R,m} \cdot k_{\text{mod}}} \tag{11}$$

with

$$z_{d,z} = \frac{\gamma_M}{f_{k,m}} \cdot (LV_G \cdot \gamma_G \cdot g_k + (1 - LV_G) \cdot \left(LV_{Q1} \cdot \gamma_Q \cdot q_{k,1} + k_m \cdot \left(1 - LV_{Q1}\right) \cdot \gamma_Q \cdot \psi_{0,2,Q2} \cdot q_{k,2} \cdot \frac{h}{b} \right)) \tag{12}$$

h/b is the cross section ratio.

2.3.4 Limit State Function and Design Parameter for Combined Bending and Compression Stress

The LSF applied for load scenario (4) and (5) is

$$g = 1 - \left(\frac{1}{z_{d,A}} \frac{\sum_i S_{c,i} \cdot \theta_{S,c,i}}{R_{c,0} \cdot \theta_{R,c,0}} + \frac{1}{z_{d,A}} \frac{\sum_i S_{m,i} \cdot \theta_{S,m,i}}{R_m \cdot \theta_{R,m}} \right) \tag{13}$$

with design parameters $z_{d,A}$ and $z_{d,M}$ from

$$0 = 1 - \left(\frac{\gamma_M \cdot \left(LV_G \cdot \gamma_G \cdot g_k + (1 - LV_G) \cdot LV_{Q1} \cdot \gamma_Q \cdot q_{k,1}\right)}{z_{d,A} \cdot k_{\text{mod}} \cdot f_{k,c}} \right)^2 - \left(\frac{\gamma_M \cdot (1 - LV_G) \cdot \left(1 - LV_{Q1}\right) \cdot \gamma_Q \cdot \psi_{0,2,Q2} \cdot q_{k,1}}{z_{d,M} \cdot k_{\text{mod}} \cdot f_{k,c}} \right) \tag{14}$$

$$0 = z_{d,M} - \frac{6 \cdot z_{d,A}}{h/b} \tag{15}$$

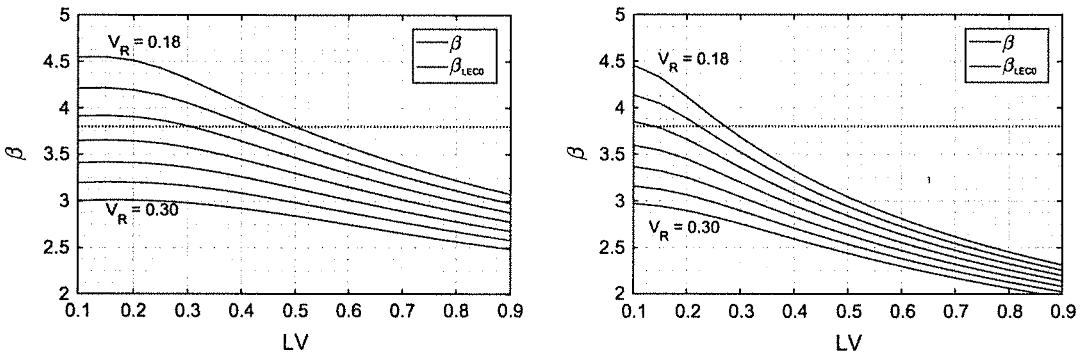

Fig. 1 Reliability index dependent on the load ratio of the live load for permanent action + live load, V_N= 0.25 (left), V_N= 0.40 (right), T_{ref}= 50a

Values for k_{mod} have been applied according to the definition in EN 1995-1-1:2010-12 [2] and National Annex, i.e. $k_{mod} = 0.8$ for live load in residence and office rooms (building occupancy type A and B), $k_{mod} = 1.0$ if wind load is acting.

3 Results

3.1 Results for Uniaxial Stress from Permanent Action and Live Load

Figure 1 illustrates that for $V_R = 0.25$ as recommended for the bending strength in [4] the target level for $T_{ref} = 50a$ β_t= 3.8 is not reached for assumptions of small (large rooms) and bigger (small rooms) live load fluctuations. For compression strength (recommended $V_R = 0.20$ in [4]) this target value is reached for live load with lower fluctuations up to a load ratio of the variable load $LV = 0.4$. As expected, greater fluctuations of live loads have a significant impact on the members reliability.

Studies resulted in relevant load rations of permanent actions and live load for common historic timber floor structures of $LV = 0.3$–0.55. Figures 2 and 3 illustrate the sensitivity factors depending on the cov of the material strength for the upper and lower bound of the load ratio and high and lower fluctuations of the live load.

3.2 Results for Bending/Two-Axial Bending from Permanent Action, Snow Load, Wind Load

Figure 4 illustrates the reliability for different load ratios of permanent action LA_G and the first variable load LA_{Q1} (snow load here) for bending and two-axial bending. Results are comparable, but the reliability decreases faster with increasing load ratio

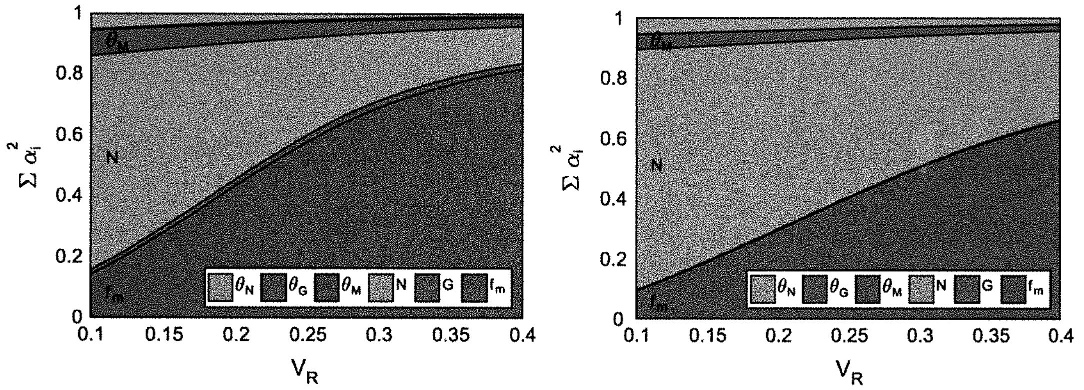

Fig. 2 Sensivity factors dependent on the coefficient of variation of the material resistance V_R for permanent action + live load, $V_N = 0.40$, $LV = 0.3$ (left), $LV = 0.55$ (right), $T_{ref} = 50a$

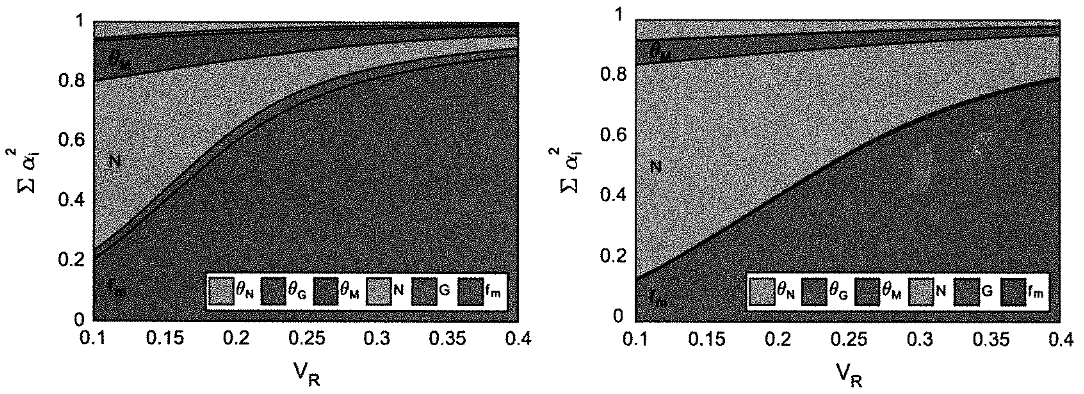

Fig. 3 Sensivity factors dependent on the coefficient of variation of the material resistance V_R for permanent action + live load, $V_N = 0.25$, $LV = 0.3$ (left), $LV = 0.55$ (right), $T_{ref} = 50a$

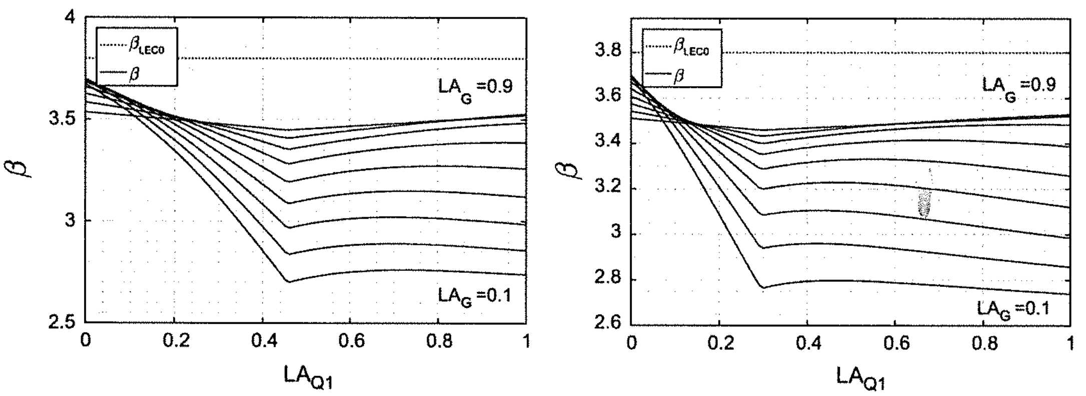

Fig. 4 Reliability index dependent on the load ratio of the first variable load for permanent action + snow load + wind load, $T_{ref} = 50a$, uniaxial bending (left), two-axial bending (right), *b/h = 1/2*

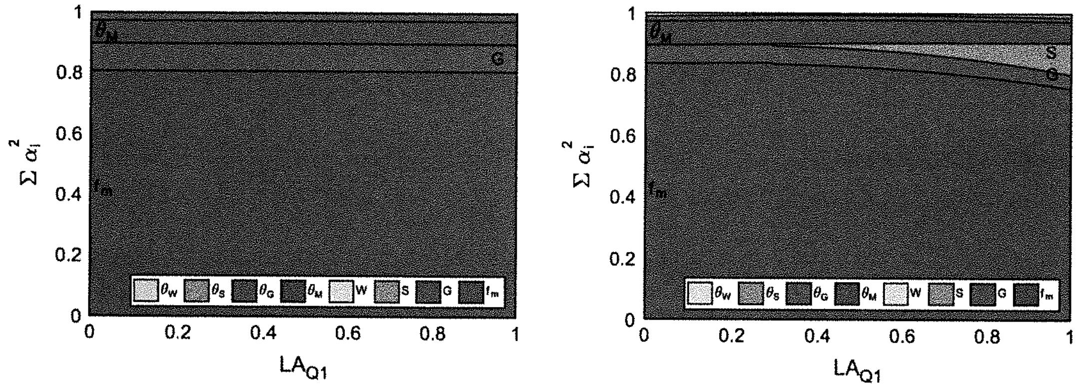

Fig. 5 Sensivity factors dependent on the load ratio for permanent action + snow load + wind load, T_{ref}= 50a, bending (left), b/h = 1/2, LA_G= 0.9 (left), LA_G= 0.7 (right)

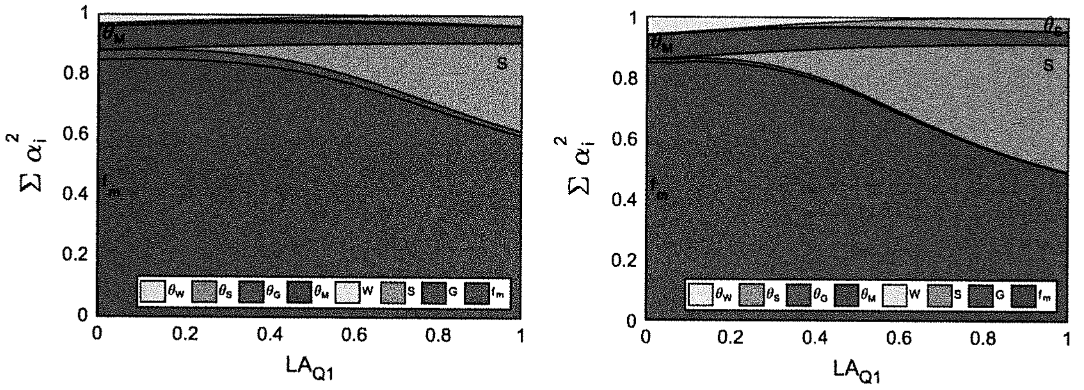

Fig. 6 Sensivity factors dependent on the load ratio for permanent action + snow load + wind load, T_{ref}= 50a, bending (left), b/h = 1/2, LA_G= 0.5 (left), LA_G= 0.3 (right)

of the snow load for two-axial bending. As the main material resistance that is activated especially in historic structures in bending strength, the material resistance has been modelled with $V_R = 0.25$. Results show, that the target value is again not reached for this load scenario. Figures 5 and 6 depict the sensitivity factors for chosen load ratios of permanent and variable actions. The figures illustrate the increasing influence of the variable loads, especially snow load, on the reliability.

3.3 Results for Bending and Compression Stress from Permanent Action, Snow Load/Live Load, Wind Load

Figure 7 shows the reliability index for bending from permanent action and snow load or live load and bending from wind load. Slenderness is not considered in these studies.

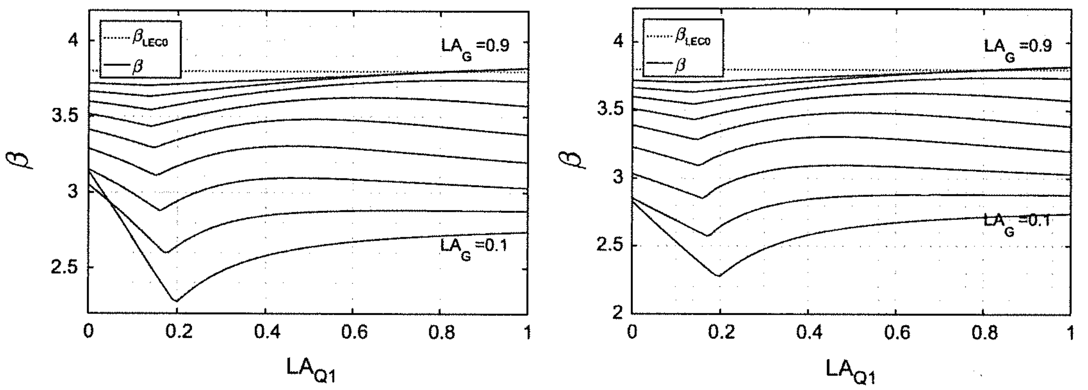

Fig. 7 Reliability index dependent on the load ratio of the first variable load for compression from permanent action + snow load (left)/ live load (right) and bending from wind load, $T_{ref}= 50a$, $b/h = 1/2$, $V_S= V_N= 0.25$

Results show that for high load ratios of permanent action, the target value $\beta_t = 3.8$ is reached. This is due to the lower variability of timber compression strength, that is activated here. For higher load ratios of variable action, the reliability decreases significantly. The point where within the semi-probabilistic design equation the second variable actions becomes the leading action can well be seen in the figures. This break in the course of the graphs results from simplifications within the semi-probabilistic model. The reliability decreases faster for the combination including snow load compared to the combination including live load.

Figures 8, 9 and 10 depict the sensitivity factors for chosen load ratios. Again, the stepwise increase of the influence of the first variable action can be seen. What is more, when superimposed with live load, the influence of wind load on the reliability seems to be greater that when superimposed with snow load. This is probably due to the assumptions of presence periods of the different types of variables actions.

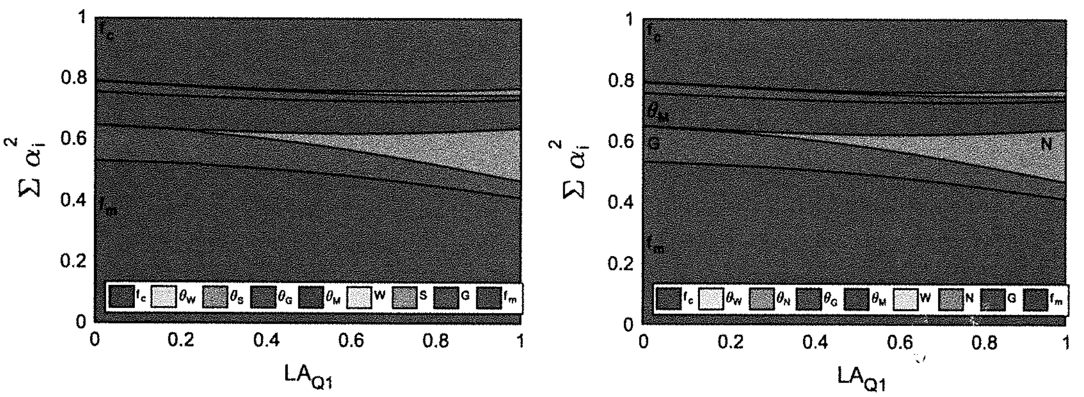

Fig. 8 Sensitivity factors dependent on the load ratio for compression from permanent action + snow load (left)/live load (right) and bending from wind load, $T_{ref}= 50a$, $b/h = 1/2$, $V_S= V_N= 0.25$, $LA_G= 0.3$

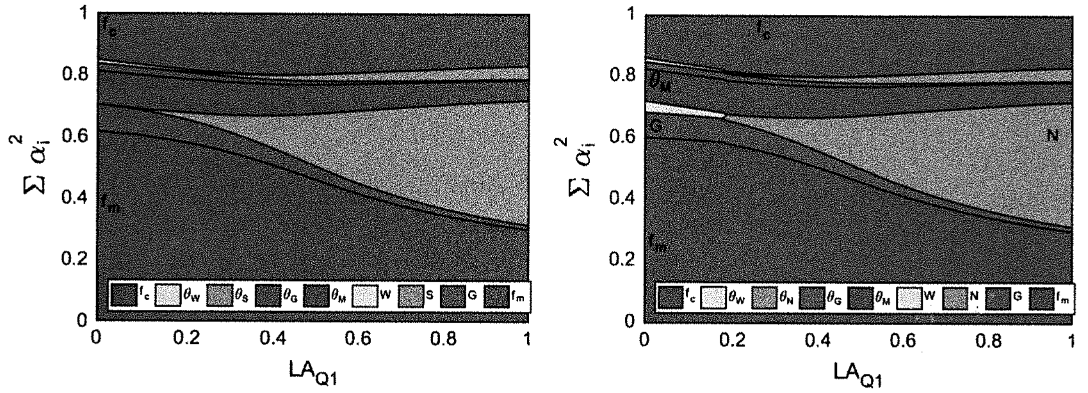

Fig. 9 Sensitivity factors dependent on the load ratio for compression from permanent action + snow load (left)/ live load (right) and bending from wind load, T_{ref}= 50a, b/h = 1/2, V_S= V_N= 0.25, LA_G= 0.5

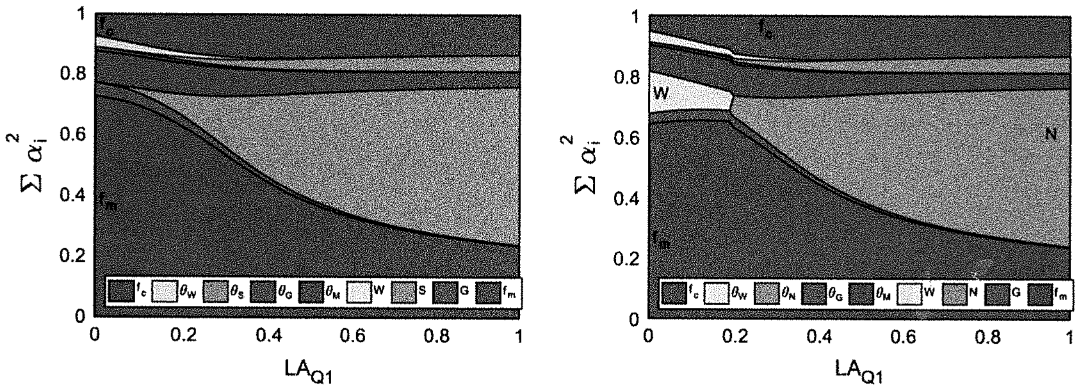

Fig. 10 Sensitivity factors dependent on the load ratio for compression from permanent action + snow load (left)/live load (right) and bending from wind load, T_{ref}= 50a, b/h = 1/2, V_S= V_N= 0.25, LA_G= 0.7

4 Conclusions

Results show, that the target value of β_t= 3.8 for T_{ref} = 50a has to be questioned. For a range of typical loading situations of timber members assuming a one-hundred percent utilization of semi-probabilistic design equations applying current partial safety factors, this value is not reached. However, this study focusses on simple limit states, more design situations have to be investigated. For example, studies on uniaxial stress from tension and bending in combination with tension showed even lower reliability indices due to the high cov of the tension strength.

For combinations with live load it could be seen that for LV = 0.3 up to V_R = 0.20 live load is dominating the reliability analysis. For LV = 0.55 and small live load fluctuations (i.e. large rooms) this is valid up to approximately V_R = 0.25 and for greater live load fluctuations (i.e. small rooms) approximately up to V_R = 0.35. Thus, for large rooms with small and higher fluctuations of live load, the reliability can be increased significantly if an investigation in situ allows for an reduction of the cov of the material strength. Compared to members subjected to bending in one direction, the reliability of members under two-axial bending decreases faster if they are designed for wind load on the weak axis and the load share of snow load on the strong axis is increased. For high load ratios of variable loads and high ratios of snow load as it is important for flat light timber roof constructions, snow load the is dominating action. Members under bending and compression show a higher reliability if load ratios of actions acting in the directions of the members axis result in higher reliability indices as the variability of the compression strength is lower compared to other timber strength properties.

To summarize, it has to be emphasized that in many cases the reliability of a timber member can be significantly increased, if a detailed survey justifies the reduction of the cov of timber material properties as this is often the dominating variable. However, individual load ratios and acting variables have to be considered carefully. What is more, the influence of members slenderness has to be studied in comprehensive analyses.

Acknowledgments The content of this contribution has been developed within funding periods of Deutsche Bundesstiftung Umwelt (AZ: 20015/409) and internal funding of Brandenburg University of Technology. The authors would like to thank the institutions for the generous support.

References

1. *Basis of structural design*, DIN EN 1990:2010-12 (2010).
2. *Design of timber structures—Part 1-1: General—Common rules and rules for buildings.* DIN EN 1995-1-1:2010-12 (2010)
3. Baravalle, M. (2017). *Risk and reliability based calibration of structural design codes.* Dissertation, Norwegian University of Science and Technology, Trondheim.
4. JCSS. (2006). *Probabilistic Model Code: Part 3—Resistance Models.* Joint Committee on Structural Safety, 2006. Available: https://www.jcss.byg.dtu.dk/. Accessed on March 08, 2016
5. JCSS. (2016). *Probabilistic model code: Part 2—Load models.* Joint Committee on Structural Safety, 2001. Available: https://www.jcss.byg.dtu.dk/Publications/Probabilistic_Model_Code. Accessed on: March 29, 2016.
6. CIB. (1989). Actions on structures: Live loads in buildings. CIB Report W81, 1989.
7. Rackwitz, R. (1996). Einwirkungen auf Bauwerke. In G. Mehlhorn (Ed.), *Der Ingenieurbau* (pp. 73–416). Berlin: Ernst.
8. Grünberg, J. (2004). *Grundlagen der Tragwerksplanung - Sicherheitskonzept und Bemessungsregeln für den konstruktiven Ingenieurbau: Erläuterungen zu DIN 1055–100* (1st ed.). Berlin: Beuth.
9. Glowienka, S. (2007). *Zuverlässigkeit von Mauerwerkswänden aus großformatigen Steinen: Probabilistische Analyse von großformatigem Mauerwerk aus Kalksandstein und Porenbeton mit Dünnbettvermörtelung.* Dissertation, Technische Universität Darmstadt (2007).
10. Vrouwenvelder, T. (2002). Developments towards full probabilistic design codes. *Structural Safety, 24,* 417–432.
11. Hansen, M. (2004). *Zur Auswirkung von Überwachungsmaßnahmen auf die Zuverlässigkeit von Betonbauteilen.* Stuttgart: Fraunhofer IRB Verlag.

Lecture Notes in Civil Engineering

José C. Matos · Paulo B. Lourenço ·
Daniel V. Oliveira · Jorge Branco ·
Dirk Proske · Rui A. Silva ·
Hélder S. Sousa *Editors*

18th International Probabilistic Workshop

IPW 2020

Springer

Lecture Notes in Civil Engineering (LNCE) publishes the latest developments in Civil Engineering - quickly, informally and in top quality. Though original research reported in proceedings and post-proceedings represents the core of LNCE, edited volumes of exceptionally high quality and interest may also be considered for publication. Volumes published in LNCE embrace all aspects and subfields of, as well as new challenges in, Civil Engineering. Topics in the series include:

- Construction and Structural Mechanics
- Building Materials
- Concrete, Steel and Timber Structures
- Geotechnical Engineering
- Earthquake Engineering
- Coastal Engineering
- Ocean and Offshore Engineering; Ships and Floating Structures
- Hydraulics, Hydrology and Water Resources Engineering
- Environmental Engineering and Sustainability
- Structural Health and Monitoring
- Surveying and Geographical Information Systems
- Indoor Environments
- Transportation and Traffic
- Risk Analysis
- Safety and Security

To submit a proposal or request further information, please contact the appropriate Springer Editor:

- Pierpaolo Riva at pierpaolo.riva@springer.com (Europe and Americas);
- Swati Meherishi at swati.meherishi@springer.com (Asia - except China, and Australia, New Zealand);
- Wayne Hu at wayne.hu@springer.com (China).

All books in the series now indexed by Scopus and EI Compendex database!

More information about this series at http://www.springer.com/series/15087

José C. Matos · Paulo B. Lourenço ·
Daniel V. Oliveira · Jorge Branco · Dirk Proske ·
Rui A. Silva · Hélder S. Sousa
Editors

18th International Probabilistic Workshop

IPW 2020

Springer